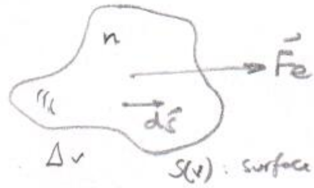
Course 9

Continuity Equations:

. need to account for carrier flow :



. $\underbrace{\text{\# electrons}}_{n}$ in a volume : $\Delta V = n . \Delta V$

$$\frac{\partial(n . \Delta v)}{\partial t} = G . \Delta v - R . \Delta v - \underbrace{\int \vec{F_e} . \vec{ds}}_{\text{sum outgoing flux through the surface of } \Delta V}$$

$$\frac{\partial n}{\partial t} = G - R - \lim_{\Delta v \to 0} \iint_{S(v)} \frac{\vec{F_e} . \vec{ds}}{\Delta v}$$

by definition:

$$\operatorname{div} F = \lim_{V \to 0} \iint_{S(v)} \frac{\vec{F} . \hat{n}}{|V|} ds \qquad \left\{ \begin{array}{l} \operatorname{div} F = \vec{\nabla} . \vec{F} \\ = \left(\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} \right) \end{array} \right.$$

$$\therefore \frac{\partial n}{\partial t} = G - R - \vec{\nabla} . \vec{F_e} \quad , \quad \vec{J_e} = -q \cdot \vec{F_e} \text{ and } \vec{J_h} = +q \cdot \vec{F_h}$$

$$\left\{ \begin{array}{l} \frac{\partial n}{\partial t} = G - R + \frac{1}{q} \cdot \vec{\nabla} . \vec{J_e} \\ \frac{\partial p}{\partial t} = G - R - \frac{1}{q} \vec{\nabla} \vec{J_h} \end{array} \right.$$

continuity equations

Quasi - neutral situation

* majority carrier concentration is close to the doping concentration

$$n(x) \sim N_D(x)$$

$$\rho \sim 0$$

$$\therefore \rho = q\left(p - n + N_D^+ - N_A^-\right) = q\underbrace{\left(p_0 - n_0 + N_D^+ - N_A^-\right)}_{\text{equilibrium carriers}} + q\cdot\underbrace{\left(p' - n'\right)}_{\text{exess carriers}}$$

· Equilibrium $\Rightarrow$ $n' = p' = 0$

· Quasi - neutrality $\Rightarrow$ $p' \simeq n'$ $\Rightarrow$ $\rho \simeq 0$
out of equilibrium

Quasi - neutrality is good if
- n, p are high
- length of problem $\gg L_D$ Debye lenght

Debye length : $L_D = \sqrt{\dfrac{\varepsilon k_B T}{q^2 N_D}}$ Scale of problem $\gg L_D$

* measure of charge carrier's net electrostatic effect and how far it persists.

$$\Rightarrow \frac{\partial E}{\partial x} = 0$$

$$E = \underbrace{E_0}_{\text{equil.}} + \underbrace{E'}_{\text{excess electric field}}$$

for which :

$$\frac{\partial E_0}{\partial x} = \frac{q}{\varepsilon}\left(p_0 - n_0 + N_D^+ - N_A^-\right)$$

and :

$$\frac{\partial E'}{\partial x} = \frac{q}{\varepsilon}\left(p' - n'\right)$$

Under quasi-neutrality, if $\rho \sim 0$, $\dfrac{\partial \rho}{\partial t} \sim 0$, thus

$$\frac{\partial J_t}{\partial x} \simeq 0$$

total current must be continuous everywhere : current continuity

⊛ two situations
- voltage is applied
- excess carriers are generated

Majority carrier situation : semiconductor resistor

- Energy given to electrons : $E_p = q \cdot V$

- Consider the case of electric field not too <u>high</u> :

$\Rightarrow$ Dynamic balance from thermal equilibrium is <u>still valid</u> :

$\Rightarrow \quad G - R = 0 \quad \Rightarrow \quad \frac{\partial J}{\partial x} = 0$

$\Rightarrow \quad n \simeq n_0$

* the function R was also treated more generally in the course by $\underline{\underline{U}}$, both meaning recombination.

$$J_e = -q n_0 \, v_e(\mathcal{E}) + q \cdot D_e \cdot \frac{dn_0}{dx}$$

In TE : $\mathcal{E} = \mathcal{E}_0 \Rightarrow J_e = 0$ (equilibrium situation)

$$J_e = -q \cdot n_0 \cdot v_e(\mathcal{E}_0) + q \cdot D_e \cdot \frac{dn_0}{dx} = 0$$

thus, out of equilibrium :

$$J_e = -q \cdot n_0 \left(\underbrace{v_e(\mathcal{E}) - v_e(\mathcal{E}_0)}_{\text{from drift}} \right)$$

, under low field $v_e = \mu_e \cdot \mathcal{E}$

$\frac{m}{s}$ $\quad$ $\frac{V}{cm}$

<u>Low-field limit :</u>

$$J_e = q \cdot n_0 \cdot \mu_e \cdot \mathcal{E}'$$

$\Rightarrow$ simplification $\Rightarrow$ no time derivatives (quasi-static situation)

<u>Integrated resistor</u>

Uniform doping $\Rightarrow \mathcal{E}_0 = 0$

Current equation :

$$J_t = -q \cdot N_D \cdot v_e(\mathcal{E})$$

for voltage not too high :

$$J_t \simeq q \cdot N_D \cdot \mu_e \cdot \mathcal{E}$$

$$J_t = \frac{I}{w \cdot t} = q \cdot N_D \cdot \mu_e \cdot \frac{V}{L}$$

(homogeneous electric field)
$$E = \frac{V}{L}$$

I-V characteristics

$$I = W \cdot t \cdot q \cdot N_D \cdot \mu_e \cdot \frac{V}{L} \Rightarrow \begin{cases} R = \dfrac{L}{q \cdot A \cdot \mu_e \cdot N_D} \\ \quad = \rho \cdot \dfrac{L}{A} \\ \rho = \dfrac{1}{q \cdot \mu_e \cdot N_D} \end{cases}$$

In general, for low and high fields:

$$I = \frac{q \cdot A \cdot N_D \cdot v_{sat}}{1 + \frac{v_{sat}}{\mu_e} \frac{L}{V}}$$

$$\therefore \quad I_{sat} = q \cdot A \cdot N_D \cdot v_{sat}$$

∴ In lateral flow in semiconductors, it is common to define a sheet resistance:

$$R = R_{sh} \cdot \frac{L}{W} \qquad R_{sh} : \Omega/\square$$

$$R_{sh} = \frac{1}{q \, \mu_e \cdot N_D \cdot t}$$

Exercise

$N_D = 10^{19}\,cm^{-3} \Rightarrow \mu_e = 110\,cm^2/Vs$

$R_{sh} = \frac{1}{q \cdot \mu_e N_D t} = 570\,\Omega/\square$

$R = R_{sh} \cdot \frac{L}{W} = 1.5K\Omega$

Minority - carrier Situation — L.Li

- excess carriers
- no significant $\vec{E}$
- quasi - neutrality

n-type : $n = n_0$

$p = p'$

recombination rate : $u = p'/\tau$

internally - generated fields are small

n type : $J_e = q(n_0 + n') \cdot \mu_e (\mathcal{E}_0 + \mathcal{E}') + q \cdot D_e \left(\frac{\partial n_0}{\partial x} + \frac{\partial n'}{\partial x} \right)$

in T.E : $n' = 0 \quad \mathcal{E}' = 0 \quad J_e = 0 \qquad q n_0 \mu_e \mathcal{E}_0 = - q \cdot D_e \frac{\partial n_0}{\partial x}$

$$J_e = \underline{q n_0 \mu_e \mathcal{E}'} + \cancel{q n_0 \mu_e \mathcal{E}_0} + \cancel{\underset{\sim 0}{q n' \mu \mathcal{E}'}} + \underline{q n' \mu \mathcal{E}_0} + \cancel{q D_e \frac{\partial n_0}{\partial x}} \Big\} \cancel{- q n_0 \mu_e \mathcal{E}_0} + \underline{q D_e \frac{\partial n'}{\partial x}}$$

majority carriers

$$J_e = \underbrace{q \cdot n_0 \mu_e \mathcal{E}'}_{\substack{\text{drift of } n_0 \\ \text{due to} \\ \mathcal{E}'}} + \underbrace{q \cdot n' \mu_e \mathcal{E}_0}_{\substack{\text{drift of } n' \\ \text{due to } \mathcal{E}_0}} + \underbrace{q \cdot D_e \frac{\partial n'}{\partial x}}_{\text{diffusion}}$$

minority carriers :

$$J_h = \underbrace{q \cdot p_0 \mu_h \mathcal{E}'}_{\simeq 0} + q p' \mu_h \mathcal{E}_0 - q \cdot D_h \cdot \frac{\partial p'}{\partial x}$$

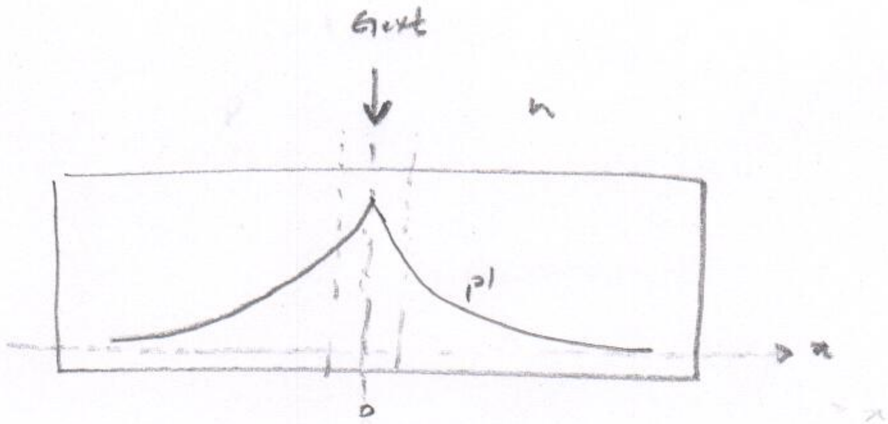
$$J_h = q p' \mu_h \mathcal{E}_0 - q \cdot D_h \cdot \frac{\partial p'}{\partial x}$$

· minority carrier continuity equation:

$$\frac{\partial p'}{\partial t} = G_{ext} - \underbrace{\frac{p'}{\tau}}_{(G-R)} - \frac{1}{q}\frac{\partial J_h}{\partial x}$$

$$\frac{\partial p'}{\partial t} = G_{ext} - \frac{p'}{\tau} - \frac{\partial p'}{\partial x}\cdot \mu_h \cdot E_0 + D_h \frac{\partial^2 p'}{\partial x^2} \qquad \left(\text{quasi neutral regions } \frac{\partial E_0}{\partial x} \approx 0\right)$$

g_e : generation of holes



out of the origin : $G_{ext} = 0$

quasi-neutral region : $E_0 \approx 0$

$$\Rightarrow D_h \cdot \frac{\partial^2 p'}{\partial x^2} - \frac{p'}{\tau} = 0$$

$x > 0$ $\quad p' = \cancel{A \exp\left(\frac{x}{L_h}\right)} + B \exp\left(-\frac{x}{L_h}\right) = B \cdot \exp^{-\frac{x}{L_h}}$

cannot be rising exponential

where $L_h = \underbrace{\sqrt{D_h \cdot \tau}}_{\text{diffusion length.}}$

$x < 0$ $\quad p' = A \exp(x/L_h)$

$B = p'(x=0) = g_e/2$

$A = p'(x=0) = g_e/2$